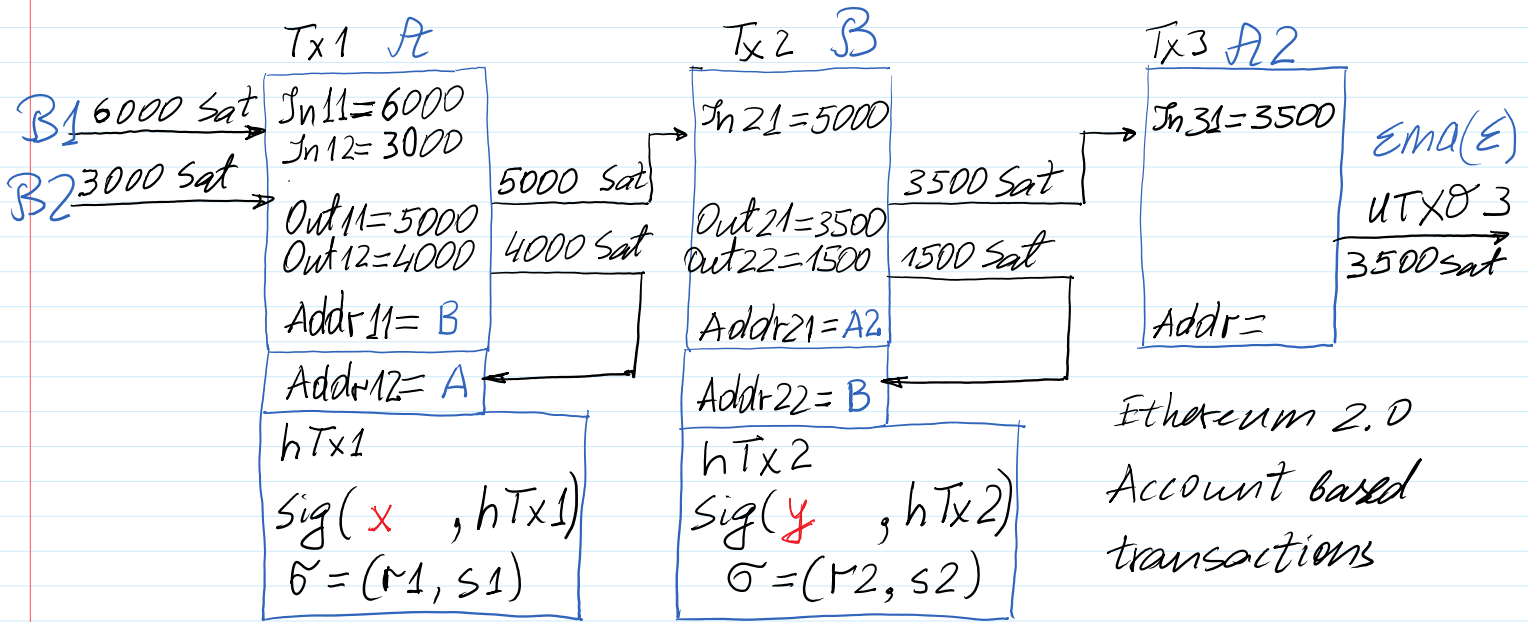


Kursinių darbų gynimas: Gruodžio 13, 20 d., 17:30 per Zoom.

Public Parameters $PP = (p, g)$; $p = 268\ 435\ 019$; $g = 2$;
 $p = \text{int64}(268435019)$
 $p = 268435019$
 $\gg g = 2$;

Unspent Transactions Output - UTXO



'Tx1: $In_{11} = 6000 \parallel In_{12} = 3000 \parallel Out_{11} = 5000 \parallel Out_{12} = 4000 \parallel Rec1 = B \parallel Rec2 = A$ '

$$hTx1 = h_{28}(\downarrow)$$

$$\text{Sign}(PrK_A = x, hTx1) = \leftarrow s_1 = \tilde{\sigma}_1 = (r_1, s_1) \quad \% \text{ Schnorr Sig.}$$

'Tx2: $In_{21} = 5000 \parallel Out_{21} = 3500 \parallel Out_{22} = 1500 \parallel Rec1 = A2 \parallel Rec2 = B$ '

$$hTx2 = h_{28}(\downarrow)$$

$$\text{sign}(PrK_B = y, h2Sig) = \leftarrow s_2 = \tilde{\sigma}_2 = (r_2, s_2) \quad \% \text{ Schnorr Sig.}$$

```
>> x=int64(randi(2^27))
x = 100497451
>> a=mod_exp(g,x,p)
a = 91968695
```

```
>> y=int64(randi(2^27))
y = 25824381
>> b=mod_exp(g,y,p)
b = 195335035
```

Schnorr Signature ← **h** - represents message to be signed
Signature is denoted by $\mathbf{S}=(r, s)$

```
u=randi(p-1)
r=gu mod p
h=hd28(M||r) % h is a decimal number
>> h=hd28(concat(M,r))
s=u+xh mod (p-1)
```

Verification of $\mathbf{S}=(r,s)$ for $\mathbf{h}=\text{hd28}(\mathbf{M}||\mathbf{r})$

$$g^s = r a^h \text{ mod } p$$

Schnorr Signature ← **h** - represents h-value to be signed in decimal format

```
Transaction Tx1 signing: h1=hd28(Tx1) = 184316888
hTx1d=hd28(Tx1)
u1=randi(p-1)
r1=mod_exp(g,u1,p)
h1=hd28(hTx1d||r1) % h1 is a decimal number
s1=u1+x(h1) mod (p-1)
```

Signature for Tx1 computation $\mathbf{S}_1=\mathbf{S1}=(r_1, s_1)$

Signature for Tx2 computation $\mathbf{S}_2=\mathbf{S2}=(r_2, s_2)$

```
>> u1=randi(p-1)
u1 = 96927099
>> r1=mod_exp(g,u1,p)
r1 = 240239134
>> h1=hd28(hTx1d || r1)
h1 = 126174618
>> xh1=mod(x*h1,p-1)
xh1 = 319146
>> s1=mod(u1+xh1,p-1)
s1 = 97246245
```

$$g^s = r a^h \text{ mod } p$$

$$g^{s_1} = (r_1) a^{h_1} \text{ mod } p$$

```

>> g_s1=mod_exp(g,s1,p)
g_s1 = 168542564
>> a
a = 91968695
>> a_h1=mod_exp(a,h1,p)
a_h1 = 128485502
>> r1
r1 = 240239134
>> r1a_h1=mod(r1*a_h1,p)
r1a_h1 = 168542564

```

```

>> hTx1d=hex2dec('AFC73D8')
hTx1d = 184316888

```

```

>> hTx2d=hex2dec('43BC2D0')
hTx2d = 71025360

```

```

%% Signature Sig1 Creation on hTx1d
Transaction Tx1 signing: M=hTx1d
u1=randi(p-1)
r1=mod_exp(g,u1,p)
h1=hd28('hTx1d||r') % h1 is a decimal number
xh1=mod(x*h1,p-1)
s1=mod(u1+xh1,p-1)

```

Sig1=(r1, s1)

Signature **Sig1** Verification

```

%% Signature verification

```

```

%% Signature Sig2 Creation on hTx2d
Transaction Tx2 signing: M=hTx2d
u2=randi(p-1)
r2=mod_exp(g,u2,p)
h2=hd28('hTx2d||r') % h1 is a decimal number
xh2=mod(y*h2,p-1)
s1=mod(u2+xh2,p-1)

```

Sig2=(r2, s2)

Signature **Sig2** Verification

```

%% Signature verification

```

Transaction template:

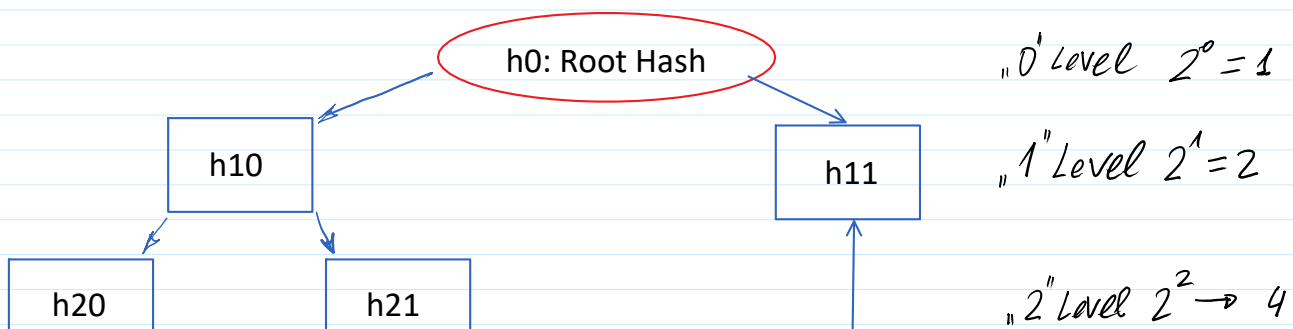
TxN = 'TxN:InN1=... | InN2=... | OutN1=... | OutN2=... | RecN1=... | RecN2=...'

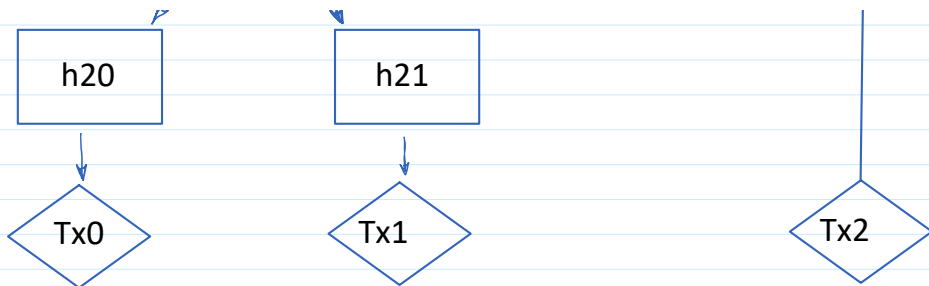
Transactions:

Tx0 = 'Tx0:In01=6000 | In02=3000 | Out01=5000 | Out02=4000 | Rec1=B | Rec2=A'

Tx1 = 'Tx1:In11=5000 | Out11=3500 | Out12=1500 | Rec1=A2 | Rec2=B'

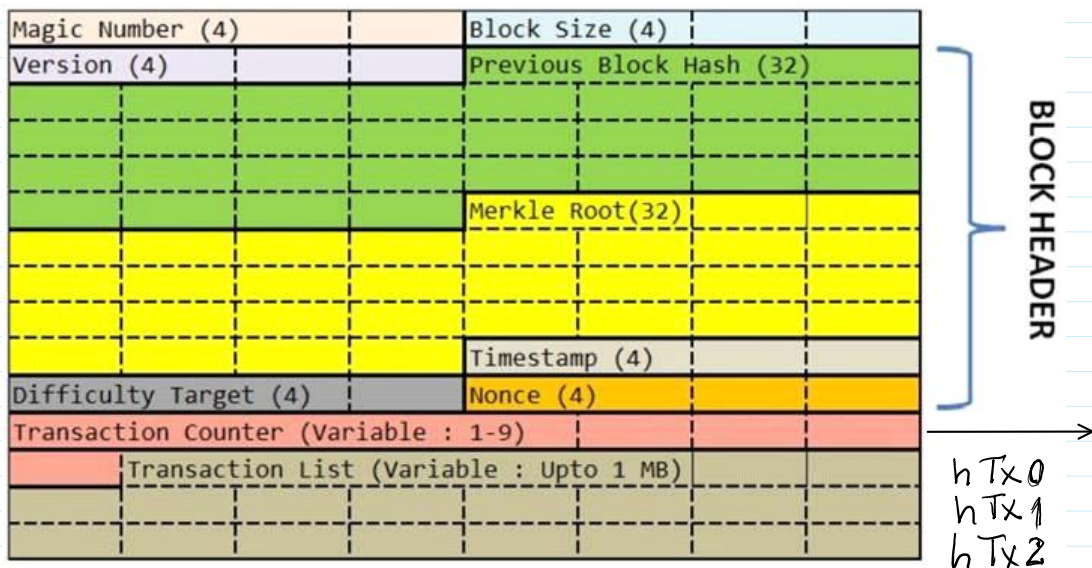
Tx2 = 'Tx2:In21=3500 | Out21=3500 | Out22=0 | Rec1=E | Rec2=0'





"2" Level $2^2 \rightarrow 4$

```
>> h20=h28(Tx0)
h20 = 8046704
>> h21=h28(Tx1)
h21 = FB4B763
>> h11=h28(Tx2)
h11 = 3619B27
>>
>> h10=h28('8046704||FB4B763')
h10 = D646CB9
>> RH=h28('D646CB9||3619B27')
RH = EFOB3B4
```



PrBlh = 0CAF06F
RH = EFOB3B4

Difficulty Target (DT): we define as 1 leading zero hex digit in block h-value.

Block data:

Bl1='Bl1:PrBlh=0CAF06F||RH=EFOB3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619B27||nonce=1000'
Bl1=h28(Bl1)
Bl1 = 676F37F

```
>> Bl1='Bl1:PrBlh=0CAF06F||RH=EF0B3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619
B27||nonce=1001'
Bl1 = Bl1:PrBlh=0CAF06F||RH=EF0B3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619B27
||nonce=1001
>> hBl1=h28(Bl1)
hBl1 = DDDDDDB6
```

Mining: Difficulty Target (DT).

```
Bl2:PrBlh=0CAF06F||RH=EF0B3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619B27||nonce=2011
nonce: 2011
```

```
Bl2: 0516A3C
```

6 hex numb. x 4 bits = 24 bits → 2^{24}

4 bits → $2^4 = 16$

```
nonce: 2235
```

```
Bl2: 00B7B95
```

```
nonce: 173147
```

```
Bl2: 0000428
```

```
nonce: 82 719 324
```

```
Bl2: 0000000
```

The number of possible h-values of 28 bits length

>> 2^{28} ans = 268 435 456

The number of adequate h-values: 2^{24}

$$\Pr\{\text{to Mine}\} = \frac{2^{24}}{2^{28}} = \frac{1}{2^4} = \frac{1}{16}$$

The number of adequate h-values: 2^{20}

$$\Pr\{\text{to Mine}\} = \frac{2^{20}}{2^{28}} = \frac{1}{2^8} = \frac{1}{256}$$

$$\Pr\{\text{to Mine}\} = \frac{2^{12}}{2^{28}} = \frac{1}{2^{16}} = \frac{1}{65536} \quad \text{: } 2^{12} \quad \text{>> } 2^{16} \text{ ans} = 65536$$

$$\Pr\{\text{to Mine}\} = \frac{1}{2^{28}} = \frac{1}{\dots} \quad \text{: } 1$$

$$Pr \{ \text{to Mine} \} = \frac{1}{2^{28}} = \frac{1}{268\,435\,456} \quad \text{ans} = 268\,435\,456$$

```
>> Bl1='Bl1:PrBlh=OCAF06F||RH=EF0B3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619
B27||nonce=1007'
Bl1 = Bl1:PrBlh=OCAF06F||RH=EF0B3B4||hTx0=8046704||hTx1=FB4B763||hTx2=3619B27
||nonce=1007
>> h=sha256(Bl1)
h = 85EDE1076C7B246DEC 0EB6B3273919E3114937AAC9BBE461A77574840A9BC018
```

"0" 184 bits

Bitcoin mining: h-value has 256 bits SHA 256
64 hex digits

Difficulty Target: to get 18 leading hex digits equal to "0".
72 leading binary digits

Total number of h-values: $2^{256} \sim 10^{80}$

Adequate number of h-values: $256 - 72 = 184 \text{ bits} \Rightarrow 2^{184}$

$$Pr \{ \text{Mining} \} = \frac{2^{184}}{2^{256}} = \frac{1}{2^{72}} ; 2^{72} =$$

```
>>> pow(2,72) = 4 722 366 482 869 645 213 696 ~ 10^21
```

```
>> 2^72 ans = 4.7224e+21
```

```
>> int64(2^72) ans = 9223372036854775807
```